RTCA Special Committee 186, Working Group 5

ADS-B UAT MOPS

Meeting #6

Simplified Equations for Modeling UAT Performance

Prepared by

Warren J. Wilson

The MITRE Corp.

SUMMARY

This paper describes some enhanced techniques for mathematically modeling UAT bit error rate performance in the presence of noise and co-channel interference.

Background

In UAT-WP-5-08, equations that fit the measured UAT performance data were provided. The equations fit the data quite well, but they involve functions (complementary error functions and hyperbolic tangents), which are computationally intensive. It may be preferable to have simpler equations for computer modeling purposes. This paper provides new (simpler) equations that fit the data for the sensitivity and co-channel measurements at least as well as the previous ones. This paper will also provide a suggestion for how to deal with cases where both noise and co-channel interference are present.

Simplified Equations

The simplified equations were found by trial and error. They have no real theoretical rationale, but they fit the data quite well in the regions where the bit error rate (BER) is between 0.1 and 0.001. These are the regions of interest, where the message success rates (MSR) go from approximately zero to approximately one.

For the sensitivity measurements the new equations are

$$BER = f(g) = \frac{1}{2}\exp\left(-Ag\,\frac{B+g}{2+g}\right) \tag{1}$$

where g is the signal-to-noise ratio (SNR) given by

$$g = \frac{D}{FkT_0W}\,. \tag{2}$$

In equation (2) D is the received signal power, F is the receiver noise figure, kT_0 is –174 dBm/Hz, and W is the receiver noise bandwidth (which is taken to be the IF filter bandwidth). For the narrow filter,

$$A = 0.1354, \quad B = 22.95$$

and for the wide filter

$$A = 0.3598, \quad B = 8.128\,.$$

When these equations (with $F = 6.5\ dB$) are used to fit the measured data from UAT-WP-4-13, the agreement is very good, as shown in figures 1 and 2. These fits are at least as good as those achieved with the more complicated expressions.

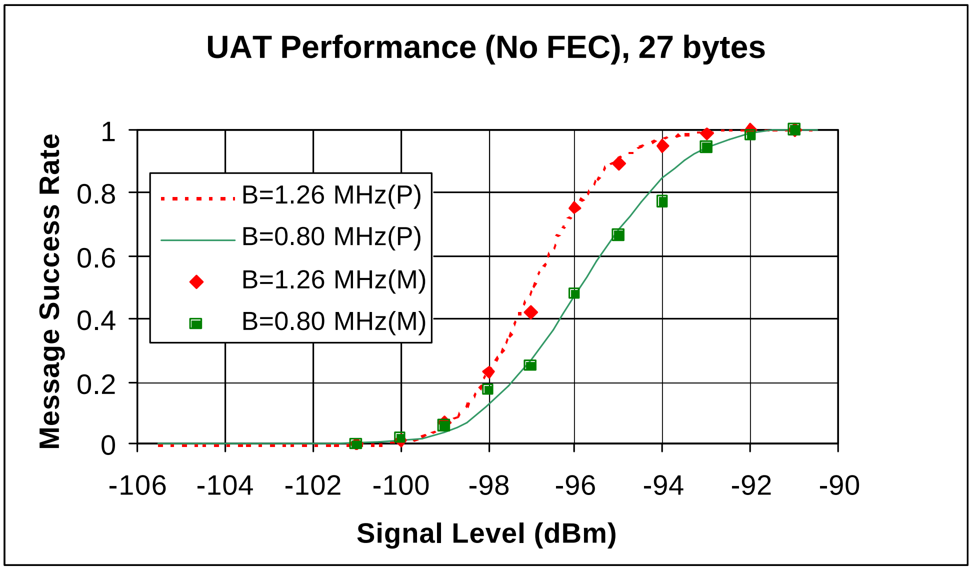

Figure 1. UAT ADS-B Performance with no FEC

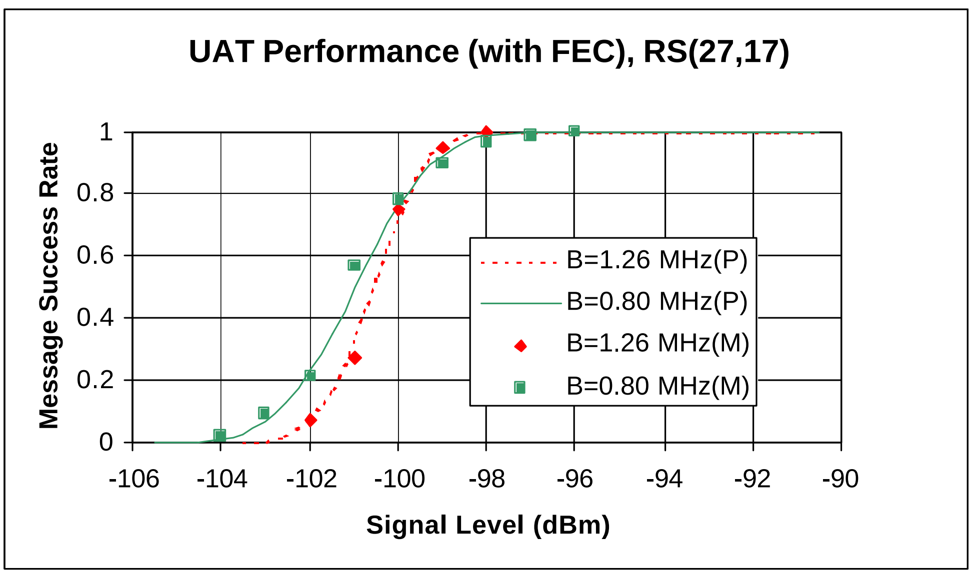

Figure 2. UAT ADS-B Performance with FEC

For co-channel performance the new equations are

$$BER = g(r) = \exp\left(-C(r-1)\right)/2 \qquad \text{if} \qquad r \geq 1 \qquad (3)$$
$$= 1/2 \qquad \text{if} \qquad r < 1,$$

where r is the desired-to-undesired ratio (D/U). For the narrow filter $C = 1.1$, and for the wide filter $C = 4.15$. Figures 3 and 4 show how these equations fit the data from UAT-WP-5-08. The fit is again at least as good as before.

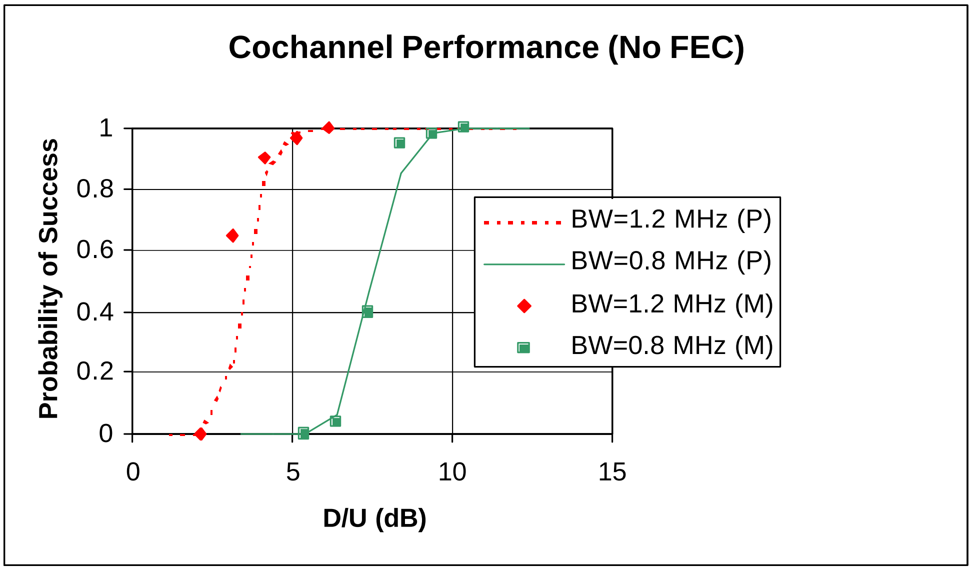

Figure 3. UAT ADS-B Co-channel Performance with no FEC (27 Bytes)

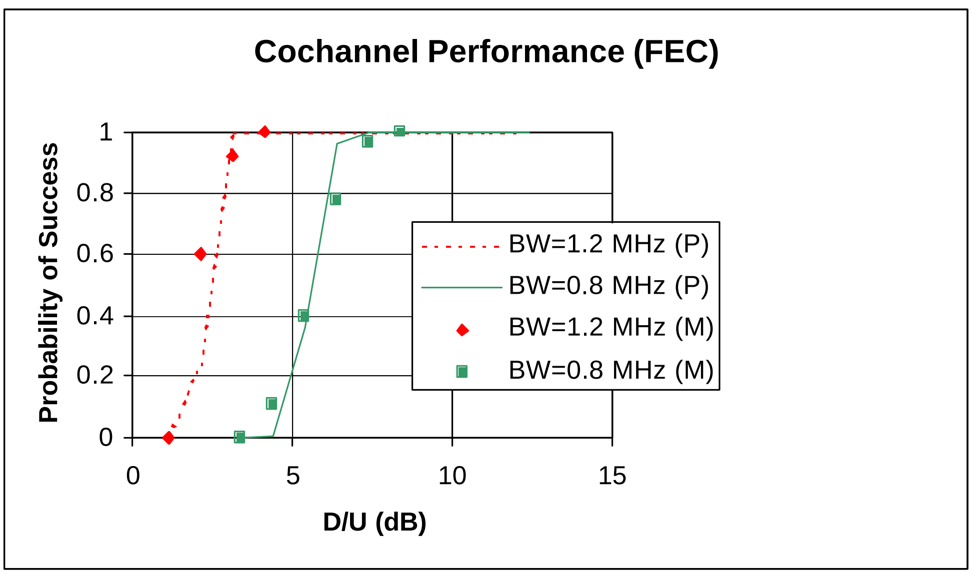

Figure 4. UAT ADS-B Co-channel Performance with FEC (RS(27,17))

Combining Noise and Co-channel Interference

Equations (1) and (3) provide good fits to the data when the interference comes from noise alone or from a single co-channel interferer. They do not provide any guidance on how to model cases where there is both noise and co-channel interference and/or multiple co-channel interferers. One possible way of combining effects is to let the BER be given by the following equation:

$$P(g,r) = f(g) + g(r) - 2f(g)g(r). \tag{4}$$

In equation (4), $f(\ )$ and $g(\ )$ are defined as before. This equation has many desirable properties:

$P(\ )$ is always between 0 and ½ as long as $f(\ )$ and $g(\ )$ are.
If either $f(\ )$ or $g(\ )$ is equal to ½ then $P(\ )$ is also equal to ½.
If $f(\ ) = 0$, then $P(\ ) = g(\ )$.
If $g(\ ) = 0$, then $P(\ ) = f(\ )$.

Thus, equation (4) is a reasonable candidate for merging the signal-plus-noise case with the desired-signal-plus-undesired-signal case.

Note that equation (4) is hardly unique. Two examples of other interpolations are:

$$P(g,r) = \frac{f(g) + g(r)}{1 + 4f(g)g(r)}$$

and

$$P(g,r) = \frac{1}{2}\left(1 - (1-2f(g))(1-2g(r))e^{-lf(g)g(r)}\right).$$

Both of these have all the desirable features mentioned above. There could be many others. Without any relevant measured data or reliable theory, it is impossible to decide which is the most accurate. Equation (4) is chosen because of its simplicity.

In more complicated situations, where the interference consists of noise plus JTIDS plus DME plus multiple co-channel interferers, a possible means of deriving the BER is to use equation (4) with g and r defined as follows:

$$g = \frac{D}{FkT_0W + J + DM + \sum_{n=2}^{N} U_n} \tag{5}$$

and

$$r = \frac{D}{U_1}. \tag{6}$$

In these equations J and DM are the powers of the JTIDS and DME pulses within the UAT receiver bandwidth, U_1 is the power of the largest co-channel interferer, and U_2 through U_N are the powers of all the other co-channel interferers. It may seem arbitrary to separate out the largest co-channel interferer in this way. This is an attempt to take into account the fact that such an interferer has different statistics than a noise source with the same average power. The constant envelope of the UAT waveform is the reason for the difference. On the other hand, multiple constant envelope signals do not have a constant envelope when they are added together. Thus, unless one co-channel interference source predominates, the error process becomes more and more noise-like as more interferers are added.

To see how this works, suppose that there are 2 equal co-channel interferers so that

$$U_1 = U_2 = U_{TOT}/2$$

and

$$D >> FkT_0W,\ D >> J,\ D >> DM\ .$$

In this case the performance estimate is given in figure 5. The curve labeled “2” shows the performance predicted from equations (4), (5) and (6). The curve labeled “1” shows the expected performance if there were only one interferer with the same total energy. The curve labeled “f” shows the expected performance if the interference were due to noise at the same power level. The figure shows that the expected performance is a blend of the two extreme curves. As the number of co-channel interferers increases, the central limit theorem indicates that the interference should be more like Gaussian noise (i.e., the curve should appear more like “f”). Figure 6, which shows the predicted performance with 4 equal cohannel interferers, indicates that this is indeed the case.

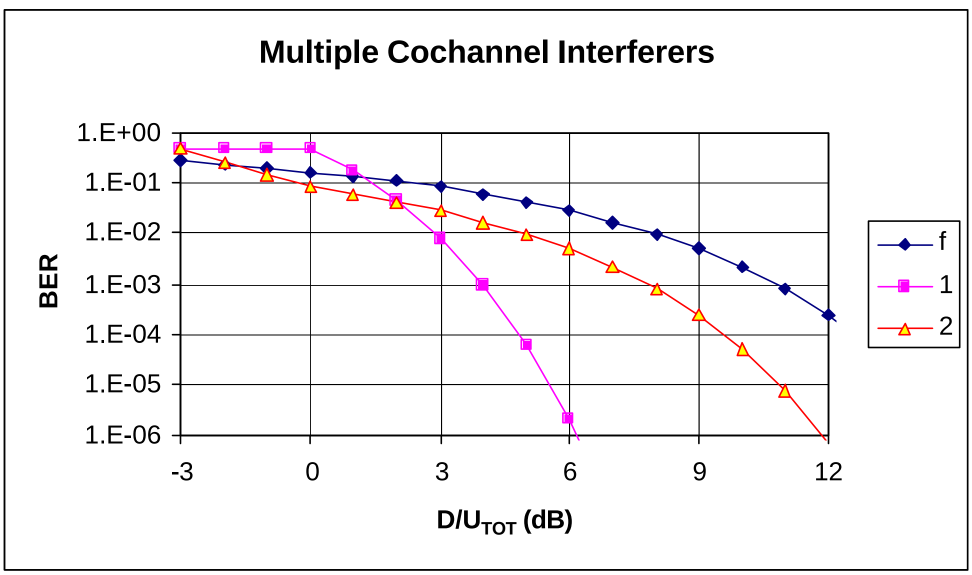

Figure 5. UAT Co-channel Performance with 2 Interferers (Wide Filter)

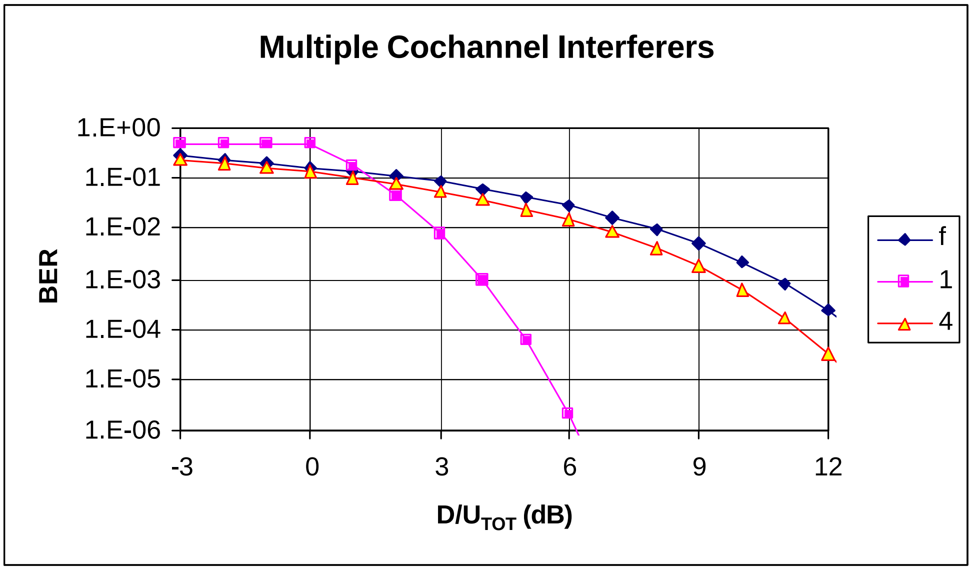

Figure 6. UAT Co-channel Performance with 8 Interferers (Wide Filter)

An Example

To show how these equations work, they were used to simulate the performance of the long ADS-B message in the LAX scenario containing 2200 potential interferers. The simulation was run (by M. Leiter of MITRE) without JTIDS interference and with the "light" and "heavy" JTIDS environments described in UAT-WP-4-04. In all cases, the ADS-B message was in the RS(48,34) format, the UAT frequency was 978 MHz, the desired UAT signal level was 25 watts, and the wider IF filter (3 dB bandwidth = 1.26 MHz) was assumed. The UAT interference was generated with the aid of the power distribution shown in figure 7, which was provided by L. Bachman of JHU APL. The equation for this curve was estimated to be

$$N = 2200\exp(-0.0046(P+104.7)^2) \qquad \text{for } P > -104.7 \qquad (7)$$
$$N = 2200 \qquad \text{for } P \le -104.7,$$

where P is the interference power measured in dBm.

The results of the simulations are shown in figures 8 and 9. In figure 8 the curve labeled (A) shows the results when using equations (4), (5) and (6) to estimate performance. The curve labeled (B) shows what the performance would have been if we had treated all of the interference as if it were noise. This shows the improvement in system performance due to the enhanced performance of the waveform in the presence of a single strong self-interference source. The improvement is quite significant.

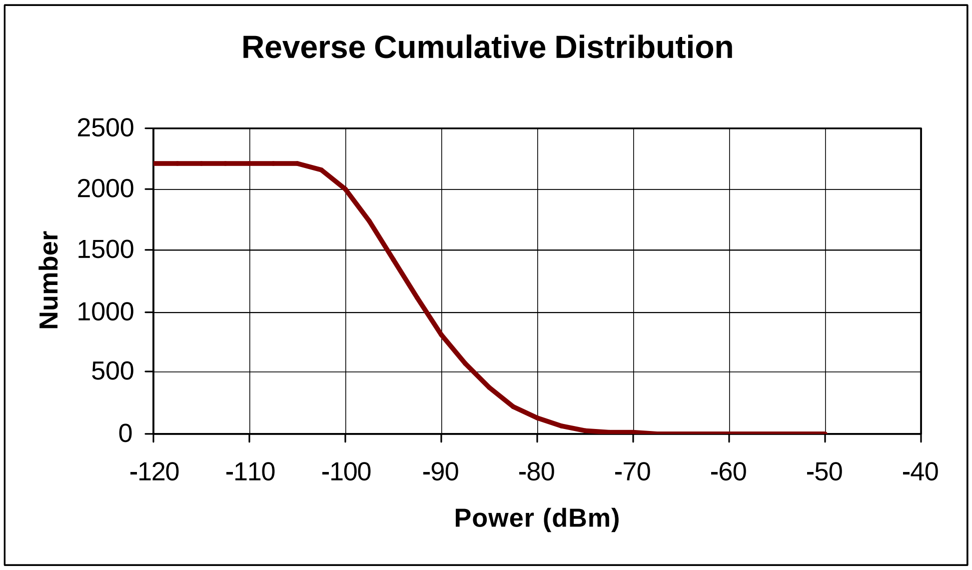

Figure 7. UAT Interference Power Distribution

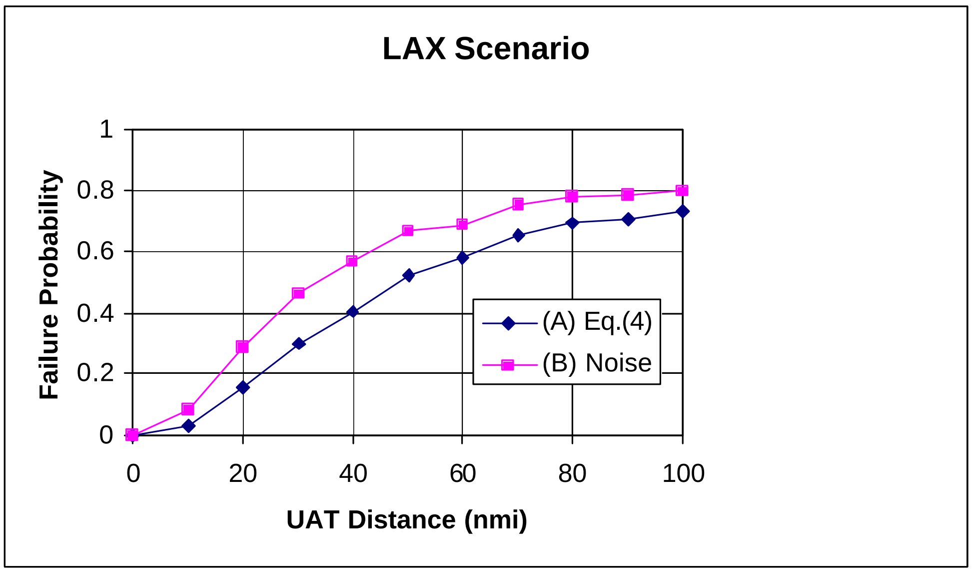

Figure 8. Simulation Results

In figure 9 curve (A) is a copy of curve (A) from figure 8. Curves (B) and (C) show the predicted performance in LAX self-interference scenario including the light and heavy JTIDS scenarios, respectively. The curves show that in this scenario, the UAT performance is dictated primarily by the co-channel performance, and the JTIDS scenarios reduce performance only marginally.

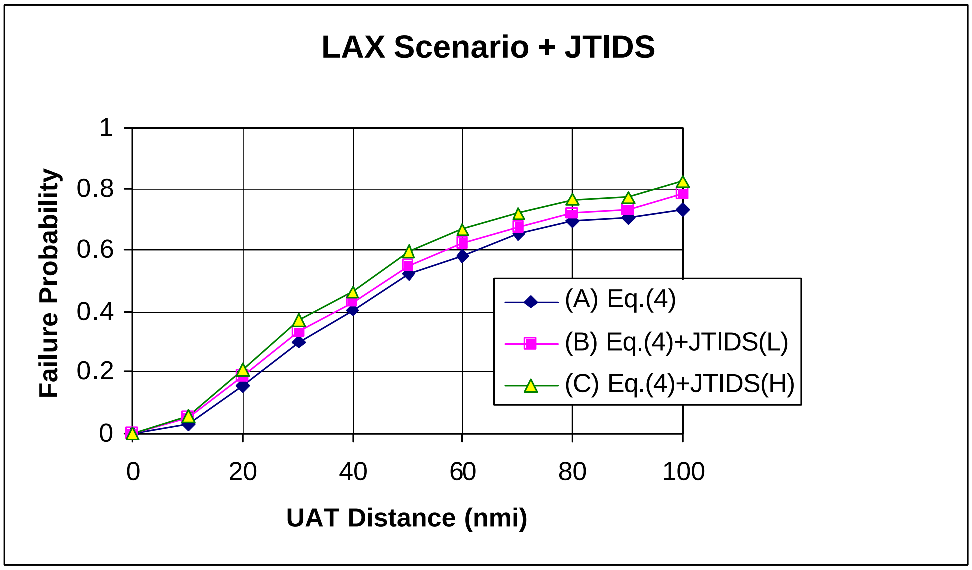

Figure 9. Simulation Results with JTIDS Interference

Additional Losses

Equation (4) is an attempt to account for the UAT performance measured in a laboratory environment. As installed, the UAT *system* may also have a cable loss of up to 3 dB plus an additional loss allowance to account for manufacturing variability. Currently, the system is specified to provide a 90% MSR for the long ADS-B message at a power level of –93 dBm as measured at the antenna output. A simple way to take these effects into account would be to increase the effective noise figure by an amount necessary to produce the specified performance.

Summary

This paper provides some *ad hoc* prescriptions for interpolating the measured UAT data in order to cover cases not directly measured but which are expected to arise in simulations and the real world. The equations have been kept as simple as possible so that if they are used in computer simulations, they will not unduly slow down the processing. In some cases (e.g., equations (4), (5), and (6)), the data are interpolated in ways that have little theoretical justification and the equations are only educated guesses. It would be useful to have measured data on some of the intermediate cases such as: cases with two co-channel interferers at various power levels, or cases with co-channel interference and the desired signal near sensitivity so that both noise and co-channel interference come into play.